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Finite Element Analysis of Box Culvert

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Abstract: The culverts are required to balance the flood water on both sides of earth embankment to reduce flood level on one side of road thereby decreasing the water head consequently reducing the flood menace. Culverts can be of different shapes such as arch, slab and box. These can be constructed with brick, stone or reinforced cement concrete. Since culvert passes through the earthen embankment, these are subjected to same traffic loads as the road carries and therefore, required to be designed for such loads. The scope of this has been further restricted to the structural design of box. The structural design involves consideration of load cases and factors like live load, effective width, dispersal of load through fill, impact factor, co-efficient of earth pressure etc. The structural elements are required to be designed to withstand maximum bending moment and shear force. So excel program is developed for analysis and it is compared with software results. So analysis of box culvert is carried out for it for various box conditions and structural design is suggested for critical cases. In skew box culvert various angles are considered and analysis of box culvert is carried out for various conditions.

Keywords: *Box Culvert, Surcharge Loading, FEM*

Introduction:

A culvert is any structure not classified as a bridge that provides an opening under a roadway, and other type of access or utility. It is well known that roads are generally constructed in embankment which comes in the way of natural flow of storm water (from existing drainage channels). As, such flow cannot be obstructed and some kind of cross drainage works are required to be provided to allow water to pass across the embankment [1]. The culvert covers up to waterways of 6 m (IRC: 5-1981) and can mainly be of two types, namely, box or slab. The box is one which has its top and bottom slabs monolithically connected to the vertical walls. In case of a slab culvert the top slab is supported over the vertical walls (abutments/ piers) but has no monolithic connection between them. A box culvert can have more than single cell and can be placed such that the top slab is almost at road level and there is no cushion. A box can also be placed within the embankment where top slab is few meters below the road surface and such boxes are termed with cushion. Culverts are provided to allow water to pass through the embankment and follow natural course of flow but these are also provided to balance the water level on both sides of embankment during floods, such culverts are termed as balancers, although there is no difference in the design. Sometimes the road alignment may cross a stream at an angle other than right angle; in such situation a skew culvert may be provided. For a smaller span there would be no difference in the design of culvert but it may require an edge beam and the layout of wing walls will have to be planned as per skew angle [2]. For a box culvert, the top slab is required to withstand dead loads, live loads from moving traffic, earth pressure on sidewalls, water pressure from inside, and pressure on the bottom slab besides self weight of the slab. The structure is designed like a rigid frame considering one meter element and adopting moment distribution method for obtaining final distributed moments on the basis of the relative stiffness of the slab and vertical walls. The method is well known and does not need any elucidation. The mid span moments are computed with free supported ends and adjusting it for moments at support obtained after distribution. The moments at center and supports for slabs and walls are obtained for various combinations of loads and the member is designed for the maximum moment it may be subjected to. Also the shear force at a distance of effective depth from the face of wall and shear stresses it produces in the section is considered in the design. It is customary to consider box a rigid frame and unit length of box is taken for design by considering the effect of all forces acting on this unit length (generally 1.0 m of box). While calculating weight of cushion on top slab, some designer take average height of earth fill coming over full length of box including sloping side fill. This is not correct and full height of cushion should be taken at the worst section of the box (central portion) will be subjected to this load and the section needs to be designed accordingly [3]. The IS: 1893-1984 (Clause 6.1.3) provide that box culverts need not be designed for earthquake forces, hence no earthquake forces are considered. Although box of maximum three cells has been discussed but in practice a box culvert can have more cells depending on the requirements at site [4].

2. Theory of culvert

2.1Introduction

It is monolithic structure having parts are top slab, bottom slab and vertical walls and wing walls. Culverts are provided to allow water to pass through the embankment and follow natural course of flow and road passes and culverts are also provided to balance the water level on both sides of embankment during floods, such culverts are termed as balancers.

There are differ types of culverts are used according to its requirement.

2.2 Types of culverts

Type of Culvert: Selecting the shape and type of culvert depends on several factors
Rigid: Reinforced concrete pipe (RCP)
Horizontal elliptical reinforced concrete pipe (HERCP)
Reinforced concrete box (RCB)
Reinforced concrete pipe arch (RCP arch)
Structural plate pipe (SPP)
Flexible: Corrugated steel pipe (CSP)
Corrugated aluminum alloy pipe (CAP)
Corrugated metal pipe (CMP)

Four-sided concrete box culverts are typically referred to as box culverts. Box culverts are drainage structures which consist of two horizontal slabs and two or more vertical walls. The slabs and walls are built monolithically. Uses of the four-sided box culvert include detention; tunnels (for conveyors, utilities, access tunnels, escape tunnels); short-span bridges (over highways, waterways, railways, golf courses); and storm drains to convey storm water, sewage, or industrial waste. Precast box culverts have gained in popularity in recent years for use as underpasses, tunnels, subways, bridges, stream culverts, material handling, storage, watertight tanks and more. Available in a variety of standard sizes as well as custom designs, box culverts can be built with features meeting the exact needs of any project: toe walls, manhole openings, headwalls, wing walls, pipe openings, keyed ends, watertight joints and more. While precast box culverts promise an extended service life.

There are three types of four-sided box culverts
1. Monolithically poured
2. Two piece mid-seam
3. Crown and base

Four-sided monolithically poured precast box culverts are poured all as one piece. The two piece mid-seam precast box culvert is a three-sided "U" and an inverted three-sided "U" joined to make a four-sided box culvert. The crown and base box culvert is yet another way to manufacture a four-sided box culvert. The crown and base box culvert is a precast inverted "U" that is joined to a concrete base[5].

3. Design loads

3.1 Introduction

The design of box culvert is based on a set of loading conditions which the component must withstand. These loads may vary depending on duration (permanent or temporary) direction of action, type of deformation ,and nature of structural action (shear, Bending ,torsion) etc.
The structural design of a reinforced concrete box culvert comprises the detailed analysis of rigid frame for moments, shear forces and thrusts due to various types of loading conditions outlined below:
1. Concentrated Loads
2. Uniform Distributed Loads
3. Weight of Side Walls
4. Water Pressure inside Culvert
5. Earth Pressure on Vertical Side Walls
6. Uniform Lateral Load on Side Walls

1. **Concentrated Loads**

In cases where the top slab forms the deck of the bridge, concentrated loads due to the wheel loads of the IRC class AA type loading have to be considered.
If P = wheel load due to IRC class AA type loading
W = Concentrated load on slab
W= (PX (I/e) kN
Where I is impact factor and e effective width of dispersion.
The soil reaction of the bottom slab is assumed to be uniform.

2. **Uniform Distributed Loads**

The weight of embankment, wearing coat and, deck slab and the track load are considered to be uniformly distributed loads on the top slab with the uniform soil reaction on the bottom slab.

3.

4. **Weight of Side Walls**

The self-weights of two side walls acting as concentrated loads are assumed to produce uniform soil reaction on the bottom slab.
Ww = is the weight of one wall
Ww = tw.H.c kN/m transversal
Where tw = wall thickness
H = height of wall, and c = density of concrete = 25 kN/m^3.

5. **Water Pressure Inside Culvert**

When the culvert is full with water, the pressure distribution on side walls is assumed to be triangular with a maximum pressure intensity of p = w.h at the base where w = density of water and h is the depth of flow.

6. **Earth Pressure on Vertical Side Walls**

The earth pressure on the vertical side walls of the box culvert is computed according to the Coulomb's Theory. The distribution of soil pressure on the side wall.

7. **Uniform Lateral Load on Side Walls**

Uniform lateral pressure on vertical side walls has to be considered due to the effect of live load surcharge. Also trapezoidal pressure distribution on side walls due to embankment loading can be obtained by combining the cases (5) and (6). Uniform lateral pressure due to the effect of surcharge loads is obtained.
Uniform lateral pressure due to the effect of surcharge loads is obtained from:

$$P= \text{surcharge loads}= \left(\frac{1-\sin\square}{1+\sin\square}\right)$$

4 Finite element methodologies

4.1Introduction

One of the main objectives of selecting a numerical model is to reduce the infinite degrees of freedom system to a limited degree of freedom, which will represent the significant physical behavior of the system. The theoretical study presented in this chapter consists of idealization of the physical system under consideration to make it amenable to treat numerically followed by selection of proper numerical technique and mathematical formulation of the specific problems.

4.2Suitability of Finite Element Method

Finite element method is a numerical analysis theory that is widely used for various situations in engineering as this method is amenable to systematic computer programming and offers scope for application to a wide range of analysis problem. Finite element analysis has now become an integral part of Computer Aided Engineering and is being extensively used in the analysis and design of many complex real-life systems. Classical analytical methods consider a differential element and develop the governing equation, usually in the form of partial differential equations. When applied to real life problem, it is often difficult to obtain an exact solution to these equations in view of complex geometry art boundary conditions. The finite element method can be viewed as a tool to transform partial differential equations into algebraic equations, which are then easily solved. Finite element method analysis allows detailed computations of the overall response in critical structural members subjected to deterministic or random loads. In vibration problem, the response is time varying and inertia/dissipation properties of the structure affect the response, the complete dynamic response analysis is usually much more complex. Finite element procedure is the best technique in this regard for the development of a simple and efficient numerical model for the solution of vibration problems directly in time and frequency domain.

4.3Theoretical Formulation

8-Noded Element

Higher order elements are useful since they utilize higher order interpolation function. Generally, the high order elements produce a more accurate solution to the differential equations. Consider an 8-node element with 2 degrees of freedom at each node that is u and v displacements. The displacement function can be written as :

$$u_e=\alpha_1+\alpha_2 x+\alpha_3 y+\alpha_4 x^2+\alpha_5 y^2+\alpha_6 x^2y+\alpha_7 xy^2$$
$$v_e=\alpha_9+\alpha_{10}x+\alpha_{11} y+\alpha_{12} x^2+\alpha_{13} y^2+\alpha_{14} x^2y+\alpha_{15} xy^2$$

At the nodes, we have :

$$ui=\alpha_1+\alpha_2 xi+\alpha_3 yi+\alpha_4 xi^2+\alpha_5 yi^2+\alpha_6 xi^2y+\alpha_7 xi\, yi^2$$
$$vi=\alpha_9+\alpha_{10}xi+\alpha_{11} yi+\alpha_{12} xi^2+\alpha_{13} yi^2+\alpha_{14} xi^2y+\alpha_{15} xi\, yi^2$$

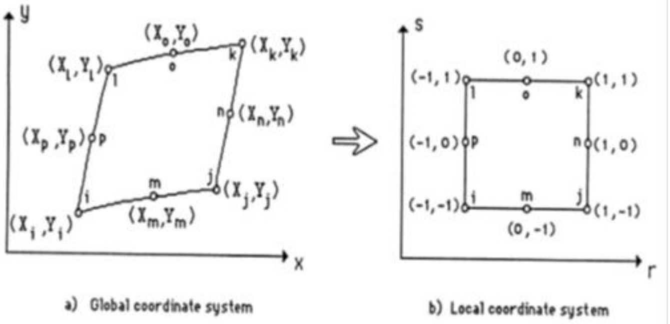

Figure *4.1 Co-ordinate system*

Equation may be written in the local coordinates with r replacing X and S replacing y. Writing Equation in terms of the shape functions and rearranging gives

$$u=N_1U_1+N_2U_2+N_3U_3+N_4U_4+N_5U_5+N_6U_6+N_7U_7+N_8U_8$$
$$V=N_1V_1+N_2V_2+N_3V_3+N_4V_4+N_5V_5+N_6V_6+N_7V_7+N_8V_8$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} N1 & 0 & N2 & 0 & N3 & 0 & N4 & 0 & N5 & 0 & N6 & 0 & N7 & 0 & N8 & 0 \\ 0 & N1 & 0 & N2 & 0 & N3 & 0 & N4 & 0 & N5 & 0 & N6 & 0 & N7 & 0 & N8 \end{bmatrix} \begin{bmatrix} U1 \\ V1 \\ U2 \\ V2 \\ U3 \\ V4 \\ . \\ . \\ U8 \\ V8 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = [N][U]$$

Where ,

$N_1=1/4\ (1-r)\ (1-s)\ (-1-r-s)$	$N_2=1/4\ (1+r)\ (1-s)\ (-1+r-s)$
$N_3=1/4\ (1+r)\ (1+s)$	$N_4=1/4\ (1-r)\ (1+s)\ (-1-r+s)$
$N_5=1/2\ (1-r2)\ (1-s)$	$N_6=1/2\ (1+r)\ (1-s2)$
$N_7=1/2\ (1-r2)\ (1+s)$	$N_8=1/2\ (1-r)\ (1-s2)$

The chain rule is used to transfer derivatives in the global coordinates into local co ordinates

$$\frac{\partial}{\partial r}=\frac{\partial}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial}{\partial y}\cdot\frac{\partial y}{\partial r}$$
$$\frac{\partial}{\partial s}=\frac{\partial}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial}{\partial y}\cdot\frac{\partial y}{\partial s}$$

And in matrix form:

$$\begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial s} \end{pmatrix} = [\,J\,] \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix}$$

Where the matrix [J] is the Jacobian matrix:

$$[\,J\,] = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix}$$

$$= \begin{bmatrix} J1 & J2 \\ J3 & J4 \end{bmatrix}$$

$$J1=\frac{\partial x}{\partial r}=Xi\ \frac{\partial N}{\partial r}\,i$$

$$J2=\frac{\partial y}{\partial r}=Yi\ \frac{\partial N}{\partial r}\,i$$

$$J3=\frac{\partial x}{\partial s}=Xi\ \frac{\partial N}{\partial s}\,i$$

$$J4=\frac{\partial y}{\partial s}=Yi\ \frac{\partial N}{\partial s}\,i$$

$$[J] = \begin{bmatrix} X\,i\frac{\partial N}{\partial r}i & Y\,i\frac{\partial N}{\partial r}i \\ X\,i\frac{\partial N}{\partial s}i & Y\,i\frac{\partial N}{\partial S}i \end{bmatrix}$$

$$= \begin{Bmatrix} \frac{\partial N}{\partial r}i \\ \frac{\partial N}{\partial s}i \end{Bmatrix} (X\,i \quad Y\,i)$$

$$=[F]\ 2X8\ \ [(X\,i \quad Y\,i)]8X2$$

$$[F]= \begin{Bmatrix} F1 \\ F2 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial N}{\partial r}i \\ \frac{\partial N}{\partial s}i \end{Bmatrix}$$

For an 8-node element, the Jacobin matrix $[J]$ can be written in the form :-

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial r} & \cdots\cdots\cdots\cdots\cdots & \frac{\partial N_8}{\partial r} \\ \frac{\partial N_1}{\partial s} & \frac{\partial N_2}{\partial s} & \frac{\partial N_3}{\partial s} & \cdots\cdots\cdots\cdots\cdots & \frac{\partial N_8}{\partial s} \end{bmatrix} \begin{bmatrix} X1 & Y1 \\ X2 & Y2 \\ X3 & Y3 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ X8 & Y8 \end{bmatrix}$$

Differentiating equation with respect to X and Y and substituting the appropriate terms yield: -

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} =[\ J\ ]^{-1}\ 2X2\ .[\ F\ ]\ 2X8\ [\ U\ ]\ 8X1$$

$$U= \begin{Bmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ \cdot \\ \cdot \\ \cdot \\ U8 \end{Bmatrix}$$

$$=[\ D\ ]\ .\ [\ U\ ] \ \text{ with }\ [\ D\ ] =[\ J\ ]^{-1}\ .[\ F\ ]$$

The same procedure can be used for S. [D] is called the derivative matrix. The stiffness matrix can be obtained from

$$\int_{-1}^{1}\int_{-1}^{1}[B]^T\,[C]\,[B]\,\det(\,J\,)\,ds\,.dr$$

$$[B] = [B1\,,B2\,,B3,\ \ldots\ldots\ldots\,.\,.\,B4]$$

5. Problem of box culvert

Design reinforced concrete box culvert having inside dimension 3m X 4.5m.The density of soil is 18 KN/m2. And subjected to the class AA tracked vehicle. Angle of repose is 30o.Adopt M-20 grade concrete and Fe 415 steel.

Clear height 3 m
Slab thickness 0.42 m
Side wall thickness 0.42 m
Unit weight of concrete 25 kN/m3
Unit weight of earth 18 kN/m3
Unit weight of water 10 kN/m3
Thickness of wearing coat 0.08 m
Concrete grade M25 = 25 Mpa
Steel grade Fe 415 = 415 Mpa
σ_{Sc} (Concrete) 8.33 Mpa
σ_{St} (Steel) 190 Mpa
Modular ratio 10
j (for effective depth) 0.902
k (for moment of resistance) 1.105 Mpa

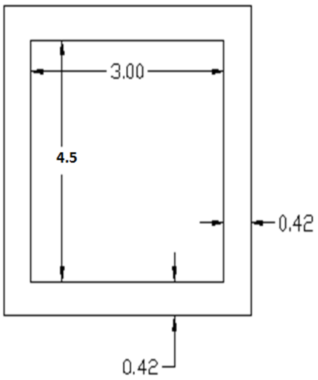

Figure 5.1 *Box Culvert (All dimension in meter)*

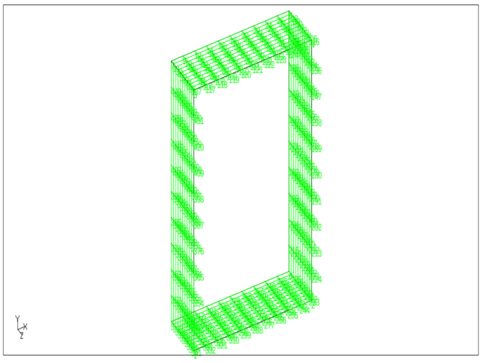

***Figure 5.2** STAAD Model of Box Culvert*

Loading Condition I A Box empty, live load surcharge on top slab of box and superimposed surcharge load on earth fill.

***Table 5.1** Comparative study of bending moments and shear force (Case I A)*

Item	Location	Member	Manual Results	Staad-Pro Results
Bending moments(kN.m)	Support	MAB,MBA, MAD, MBC	-57.92	-51.22
		MDC,MCD, MDA, MCB	85.53	80.87
	Mid span	MAB, MBA MDC, MCD, MAD, MBC	31.98 48.54 -57.92	28.23 45.89 -54.92
Shear Forces(kN)	At d_{eff} from top slab for wall	A & B	105.14	107.98
	At d_{eff} from bottom slab for wall	D & C	156.80	155.34

5.1 Loading Condition I B

5.1.1 Box inside full with water, live load surcharge on top slab and superimposed surcharge load on earth fill.

***Table.5.2** Comparative study of bending moments and shear force (Case I B)*

Item	Location	Member	Manual Results	Staad-Pro Results
Bending moments(kN.m)	Support	MAB,MBA, MAD, MBC	-37.26	-31.11
		MDC,MCD, MDA, MCB	71.29	67.87
	Mid span	MAB, MBA MDC, MCD, MAD, MBC	52.64 62.78 -24.20	49.23 55.89 -24.92
Shear Forces(kN)	At d_{eff} from top slab for wall	A & B	105.14	107.98
	At deff from bottom slab	D & C	156.80	155.34

5.1.3 Loading Condition II A

Box empty with water, live load surcharge on top slab and no superimposed surcharge on earth fill.

***Table.5.3** Comparative study of bending moments and shear force (Case II A)*

Item	Location	Member	Manual Results	Staad-Pro Results
Bending moments(kN.m)	Support	MAB,MBA, MAD, MBC	-47.21	-45.67
		MDC,MCD, MDA, MCB	77.21	73.43
	Mid span	MAB, MBA MDC, MCD, MAD, MBC	42.70 56.86 -47.20	38.87 54.21 -45.99
Shear Forces(kN)	At d_{eff} from top slab for wall	A & B	105.14	107.98
	At d_{eff} from bottom slab for wall	D & C	156.80	155.34

5.1.4 Loading Condition II B

Box inside full with water, live load surcharge on top slab and no superimposed surcharge on earth fill

Table.5.4 *Comparative study of bending moments and shear force (Case II B)*

Item	Location	Member	Manual Results	Staad-Pro Results
Bending moments(kN.m)	Support	MAB,MBA, MAD, MBC	-59.38	-55.87
		MDC,MCD, MDA, MCB	25.35	23.68
	Mid span	MAB, MBA MDC, MCD, MAD, MBC	64.54 74.68 -39.70	62.81 72.84 -31.34
Shear Forces(kN)	At d_{eff} from top slab for wall	A & B	105.14	107.98
	At d_{eff} from bottom slab for wall	D & C	156.80	155.34

Critical cases of bending moments (kN.m)

*“**Table .5.5** Study of critical cases of bending moments”*

Water Condition	Lateral Uniform Loading	Top Slab		Bottom Slab		Vertical Wall	
		centre	end	centre	end	centre	end
EMPTY	(D.L.+L.L.)/3	31.98	-57.92	48.54	85.53	-57.92	85.53
FULL	(D.L.+L.L.)/3	52.64	-37.26	62.78	71.29	-24.20	71.29
EMPTY	(D.L.) /3	42.69	-47.21	56.80	77.21	-47.20	77.21
FULL	(D.L.)/3	64.54	-59.38	74.68	25.35	-39.70	25.35

6. Conclusion

In the present report, analysis of a Reinforced concrete box culvert has been presented using finite element method. Three dimensional configuration of the space has been considered and computer code has been developed for finding the bending moments, member forces and support reactions due to equivalent traffic load, lateral soil pressures. Equivalent moments, member forces and support reactions are calculated by excel programming. For different conditions analysis is carried out and design is suggested for the critical cases. Some of the conclusions arrived at the present study are as follows

1. Box full with water, live load surcharge on top slab of box condition then the bending moments at centre and end of the top and bottom slab are increased.
2. Box empty, live load surcharge on top slab of box condition, vertical wall bending moment is increased at centre and end.
3. The design of box is covered by four load cases. The situation when Box inside full with water, provide design moments less than given by the other load cases.
4. Box empty, live load surcharge on top slab of box and superimposed surcharge load on earth fill condition gives maximum moments. it is critical case but the design values should be taken the maximum bending moment among the all conditions.

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